# Square lattice representations of P, D, and G surfaces and their mixtures and generalizations

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We present a constructive mechanism for the P, D, and G triply periodic minimal surfaces and their hybrids by utilizing a decomposition reflecting the tetragonal lattice symmetry. In this context, the surfaces can be seen as filling in a square lattice with infinitely long 1D strips (with tetragonal boundary condition), where P strips are disjoined catenoids vertically stacked, D strips twisted helicoids, and G strips somewhere in between. This is reminiscent of the catenoid-helicoid transformation and/or the Bonnet transformation. Except for the P surface, both G and D strips trace out a helical boundary curves on the surface of the tetragonal cell and are chiral. By assuming glide symmetry planes along the grids of the square lattices, one recovers the original Schwarz P, D and Schoen's G surfaces. See Figure 1.

It becomes rather clear that such decomposition allows one to mix D and G strips together and form a hybrid D/G surface. It is unclear that such topology allows a minimal surface. However, since they share the same tetragonal boundary condition, one can create a seamless transition between a purely D compartment and a purely G one, essentially forming a grain boundary between the two semi-infinite surfaces. The 2D unit cell can be arbitrarily large with arbitrarily large genus.

By adjusting the boundary curve of P strips, referred to as a P' strip, one can also seamlessly join them with D or G ones and form hybrid surfaces/grain boundaries. A 3D cubic lattice generalization is possible but will be left for future exploration.

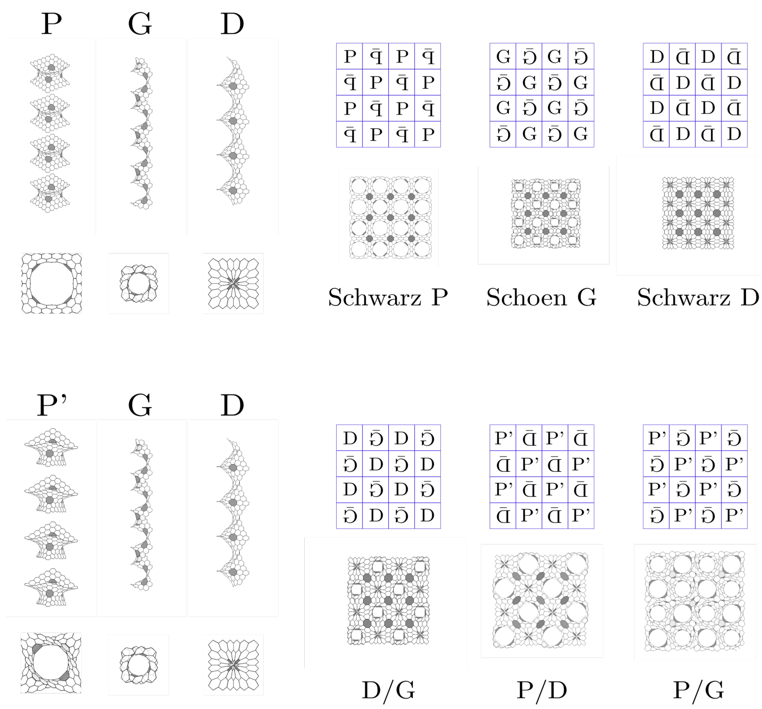


**Figure 1** (Top left) The P, G, and D strips. (Top right) Square lattices filled with single type strips with glide plane symmetry. (Bottom left) The P', G, and D strips. (Bottom right) Mix-matched square lattices giving rise to hybrid surfaces. The barred characters refer to glide-symmetric (reflected in the xz/yz plane and translated half a spacing in the z direction.) Here we utilize a graphitic structure representation where each vertex (sp2 carbon atom) is connected to three nearest neighbors.

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[1] B.-Y. Jin, C. Chuang, C.-C Tsoo, *Proceedings of Bridges: Mathematics, Music, Art, Architecture, Culture*, 503-506 (2012).
[2] C. Chuang, B.-Y. Jin, *J. Math. Chem.*, **47**, 1077-1084 (2010).