Triply periodic minimal surfaces revisited: surface complexes

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Alan Schoen made many contributions to physics and geometry beyond the Gyroid, some of which have been described in a recent obituary (ref. 1). The Gyroid, like Schwarz' P and D triply-periodic minimal surfaces (TPMS), define a pair of catenated 3d labyrinths and are *bicontinuous*. These surfaces partition 3d euclidean space into *two* extended three-periodic open cells, bounded by the TPMS. In that sense they generalise simpler classical space partitions, namely the tessellation of 3d euclidean space by an *infinite* number of plane-faced polyhedra, e.g. congruent cubes or "saddle polyhedra" - whose faces are curved minimal surfaces - explored by the architect Paul Pearce (ref. 2).

Both labyrinths of TPMS were perceived by Schoen as inflated three-periodic skeletal nets (or "labyrinth graphs"). By analogy saddle polyhedral cells, which are topological balls, are zero-periodic. These examples of cellular decompositions of space suggest more general lines of inquiry. If we constrain the cells to be bounded by faces which are minimal surfaces:

(i) can we find partitions of of 3d euclidean space beyond zero- and three-periodic cases?

(ii) can space be partitioned into z equal cells, apart from z=2 (TPMS) or z=∞ (polyhedral tessellations)?

The answers are yes and yes. Simpler cases are self-intersecting TPMS, many of which Schoen explored (in ref. 3), succeeded by the crystallographic studies of Fischer and Koch (e.g. refs. 4, 5). Later on, we found a number of three-periodic minimal *surface complexes (TPMS*)*, which include three-fold branched lines (e.g. ref. 6) which include novel *polycontinuous* partitions for molecular self-assemblies (ref. 7).

In fact, Schoen had explored TPMS* earlier, which he called "integral varifolds" (ref. 3). On meeting Schoen in 2011, all three authors of this paper decided to revisit his earlier study. We have reanalysed a complete suite of minimal surface Flächenstücke bounded by skew quadrilaterals which coincide with two-, three (and four-) fold axes of rotational symmetry of the cube. Among many more cases, twelve TPMS* with cubic symmetry emerge, containing zero-, one-, two- and three-periodic cells, afford a rich inventory of partitions satisfying both (i) and (ii) above. Interesting examples - some of which have been derived independently by Longdell, Oguey and Hyde (unpublished) - include TPMS* whose skeletal nets describe well-known crystallographic rod-packings and novel mutually catenated inclined meshes. A tetracontinuous TPMS* contains a quartet of like-handed *srs* skeletal nets (in contrast to the enantiomeric pair of srs nets characterising the Gyroid). The most spectacular TPMS* defies all intuition: it partitions 3d space in to a *single* three-periodic cell!

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