Polycontinuous pattern and 3-dimensional topology

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In this talk, we explore the concept of handlebody decomposition of 3-manifolds, extending beyond the well-known Heegaard splittings and trisections. A handlebody decomposition divides a closed orientable 3-manifold into several handlebodies. Our main result demonstrates that two handlebody decompositions of a closed orientable 3-manifold are stably equivalent, meaning they become isotopic after a finite number of stabilizations. This generalization provides a new perspective on the structural properties of 3-manifolds.

We apply this theoretical framework to materials science by modeling polycontinuous patterns, which arise in the microphase separation of block copolymer melts. These patterns can be viewed as triply periodic non-compact surfaces, tribranched surfaces, or polyhedra embedded in R^3, dividing it into multiple submanifolds. Our model characterizes bicontinuous, tricontinuous, and polycontinuous patterns, which are of significant interest for understanding the morphology of copolymer systems.

A key aspect of our study is the correspondence between triply periodic polycontinuous patterns and handlebody decompositions of the 3-dimensional torus T^3. We show that such patterns, under suitable conditions, induce handlebody decompositions of T^3. Consequently, the Reidemeister-Singer-type theorem for handlebody decompositions extends to polycontinuous patterns, offering insights into their structural equivalence and transformation.

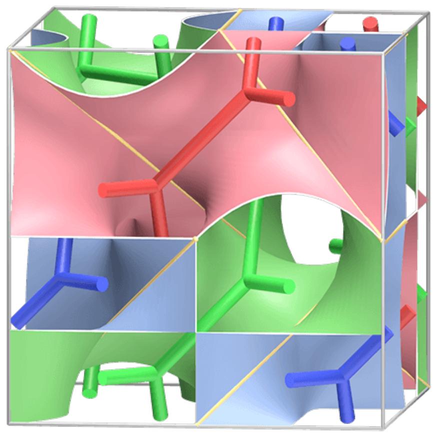

Figure 1 Nets and tricontinuous pattern.

Through this topological approach, we provide a mathematical foundation for the classification and analysis of complex copolymer structures, highlighting the interplay between 3-manifold topology and material science. Our results not only advance the theoretical understanding of 3-manifolds but also have practical implications for designing novel materials with desired mechanical properties.

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[1] N. Sakata, R. Mishina, M. Ogawa, K. Ishihara, Y. Koda, M. Ozawa, and K. Shimokawa, Proc. R. Soc. A 478: 20220073 (2022).