Exploring pattern formation on negatively curved surfaces via the hyperbolic plane

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We investigate the self-assembly behaviour of block copolymers constrained to thin hyperbolic films. Specifically, we study the pattern formation on the three-periodic cubic minimal surfaces P, D and G found ubiquitously in soft matter material science. We use a new method for visualisation and analysis of the patterns by mapping to two-dimensional hyperbolic space analogous to stereographic projections in cartography thus effectively creating a more accessible "hyperbolic map" of the pattern [1]. This allows us to pinpoint in detail the role of intrinsic geometry and to probe the role of negative curvature on the resulting assemblies as contrasted with flat and positively curved films. We present results from AB diblock copolymers and ABC mikto-arm star terpolymers of varying composition. In the case of compositionally balanced diblocks, the resulting patterns are related to ``free" tilings of the hyperbolic plane [2,3] while unbalanced molecules form a plethora of disc packings. Star polymers form three-colored cellular patterns related to curved graphene-like schwarzites constrained to only form even polygons.

Keywords: block copolymers, self-assembly, triply periodic minimal surfaces, hyperbolic geometry.

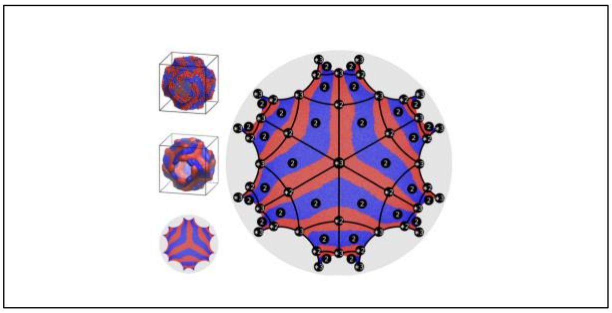

Figure1. *AB diblock copolymers self-assembling on the P-surface and mapped to the hyperbolic plane for analysis [1].*

References

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